Chapter 9. Principles of Electromechanical Energy Conversion

Topics to cover:
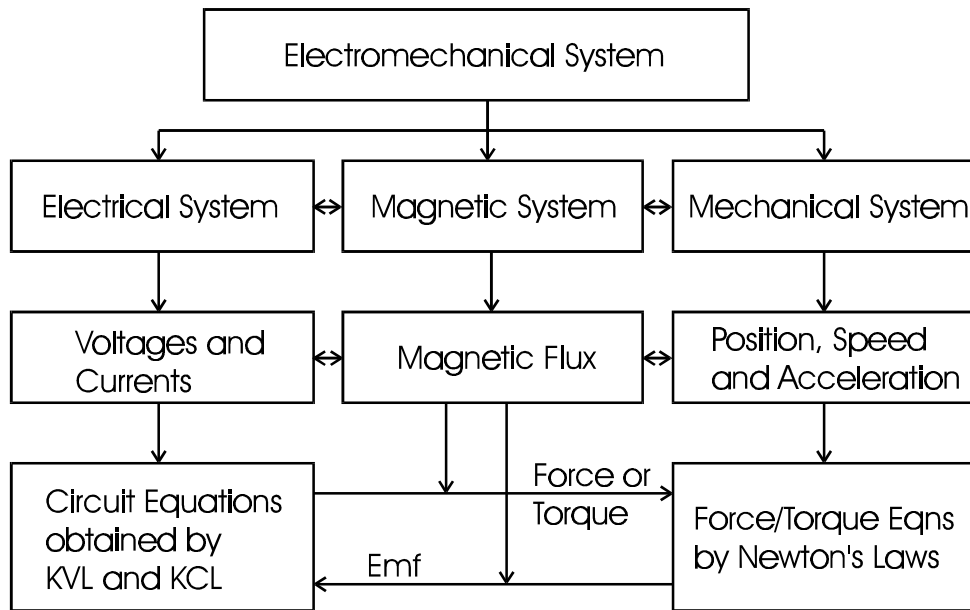
1) *Introduction*
2) *EMF in Electromechanical Systems*
3) *Force and Torque on a Conductor*
4) *Force and Torque Calculation from Energy and Coenergy*
5) *Model of Electromechanical Systems*

Introduction

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories:

(1) Transducers (for measurement and control)
   These devices transform the signals of different forms. Examples are microphones, pickups, and speakers.
(2) Force producing devices (linear motion devices)
   These type of devices produce forces mostly for linear motion drives, such as relays, solenoids (linear actuators), and electromagnets.
(3) Continuous energy conversion equipment
   These devices operate in rotating mode. A device would be known as a generator if it convert mechanical energy into electrical energy, or as a motor if it does the other way around (from electrical to mechanical).

Since the permeability of ferromagnetic materials are much larger than the permittivity of dielectric materials, it is more advantageous to use electromagnetic field as the medium for electromechanical energy conversion. As illustrated in the following diagram, an electromechanical system consists of an electrical subsystem (electric circuits such as windings), a magnetic subsystem (magnetic field in the magnetic cores and airgaps), and a mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator and a rotor in a rotating electrical machine). Voltages and currents are used to describe the state of the electrical subsystem and they are governed by the basic circuital laws: Ohm's law, KCL and KVL. The state of the mechanical subsystem can be described in terms of positions, velocities, and accelerations, and is governed by the Newton's laws. The magnetic subsystem or magnetic field fits between the electrical and mechanical subsystems and acting as a "ferry" in energy transform and conversion. The field quantities such as magnetic flux, flux density, and field strength, are governed by the Maxwell's equations. When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movement of the moving part will could variation of the magnetic flux linking the electric circuit and induce an electromotive force (*emf*) in the circuit. The product of the torque and speed (the mechanical power) equals the active component of the product of the *emf* and current. Therefore, the electrical energy and the mechanical energy are inter-converted via the magnetic field.

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Concept map of electromechanical system modeling

In this chapter, the methods for determining the induced *emf* in an electrical circuit and force/torque experienced by a movable part will be discussed. The general concept of electromechanical system modeling will also be illustrated by a singly excited rotating system.

Induced *emf* in Electromechanical Systems

The diagram below shows a conductor of length *l* placed in a uniform magnetic field of flux density **B**. When the conductor moves at a speed **v**, the induced *emf* in the conductor can be determined by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

The direction of the *emf* can be determined by the "right hand rule" for cross products.

In a coil of *N* turns, the induced *emf* can be calculated by

$$e = -\frac{d\lambda}{dt}$$

where λ is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement.

In practice, it would convenient if we treat the *emf* as a voltage. The above express can then be rewritten as

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self ***inductance is a function of the displacement*** x since there is a moving part in the system.

Example:

Calculate the open circuit voltage between the brushes on a Faraday's disc as shown schematically in the diagram below.
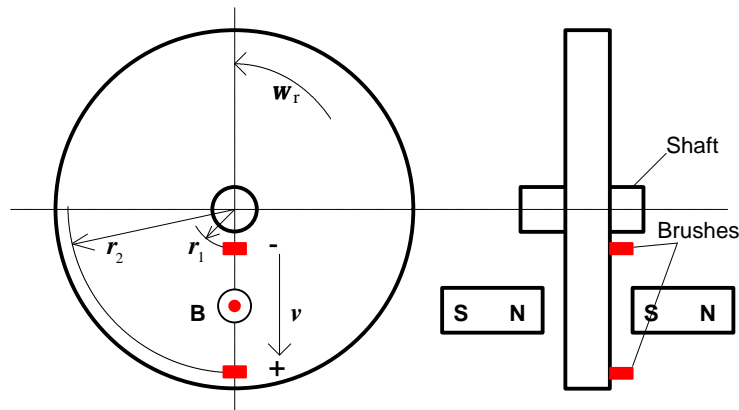
Solution:

Choose a small line segment of length dr at position r (r_1£r£r_2)from the center of the disc between the brushes. The induced emf in this elemental length is then

$$de = Bvdr = B\mathbf{w}_r rdr$$

where v=r$\mathbf{w}_r$. Therefore,

$$e = \int de = \int_{r_1}^{r_2} B\mathbf{w}_r rdr = \mathbf{w}_r B \frac{r^2}{2}\bigg|_{r_1}^{r_2} = \mathbf{w}_r B \frac{r_2^2 - r_1^2}{2}$$

Example:

Sketch L(x) and calculate the induced emf in the excitation coil for a linear actuator shown below.
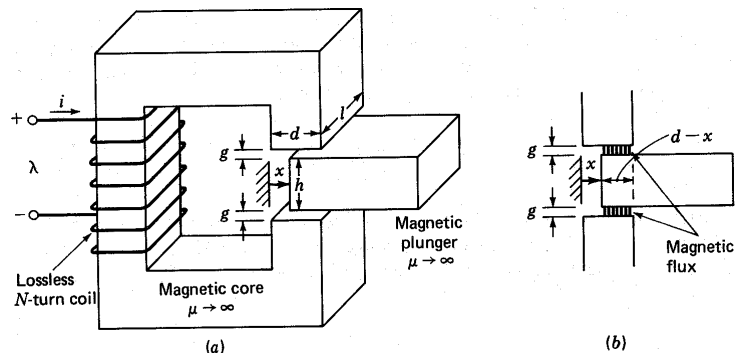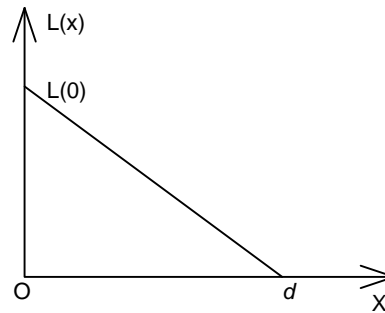

A singly excited linear actuator

Solution:

$$L(x) = \frac{N^2}{R_g(x)}$$

and

$$R_g(x) = \frac{2g}{\mu_o(d-x)l}$$

$$\therefore L(x) = \frac{\mu_o N^2 l}{2g}(d-x)$$

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

$$= L(x)\frac{di}{dt} - i\frac{\mu_o N^2 l}{2g}v$$

Inductance vs. displacement

If i=I$_{dc}$,

$$e = -I_{dc}\frac{\mu_o N^2 l}{2g}v$$

If i=I$_m$sinωt
,

$$e = \frac{\mu_o N^2 l}{2g}(d-x)\omega I_m \cos\omega t - vI_m \sin\omega t\frac{\mu_o N^2 l}{2g}$$

$$= I_m\frac{\mu_o N^2 l}{2g}\left[(d-x)\omega\cos\omega t - v\sin\omega t\right]$$

$$= I_m\frac{\mu_o N^2 l}{2g}\sqrt{(d-x)^2\omega^2 + v^2}\cos\left[\omega t + \arctan\left(\frac{v}{(d-x)\omega}\right)\right]$$

Force and Torque on a Current Carrying Conductor

The force on a moving particle of electric charge q in a magnetic field is given by the Lorentz's force law:

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

The force acting on a current carrying conductor can be directly derived from the equation as

$$\mathbf{F} = I\int_C d\mathbf{l} \times \mathbf{B}$$

where *C* is the contour of the conductor. For a homogeneous conductor of length *l* carrying current *I* in a uniform magnetic field, the above expression can be reduced to

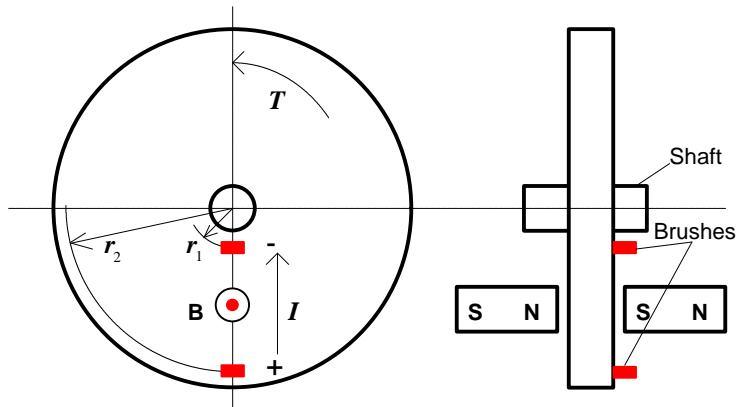$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$$

In a rotating system, the torque about an axis can be calculated by

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

where **r** is the radius vector from the axis towards the conductor.

Example:

Calculate the torque produced by the Faraday's disc if a dc current I_{dc} flows from the positive terminal to the negative terminal as shown below.

Solution:

Choose a small segment of length dr at position r (r_1£r£r_2) between the brushes. The force generated by this segment is

$$d\mathbf{F} = (-Idr\hat{\mathbf{r}}) \times (B\mathbf{a}_z)$$
$$= IBdr\mathbf{a}_q$$

*where $\mathbf{a}_q$ is the unit vector in **q** direction. The corresponding torque is*

$$d\mathbf{T} = \mathbf{r} \times d\mathbf{F} = IBrdr\mathbf{a}_z$$

Therefore,

$$T = \int d\mathbf{T} = \int_{r_1}^{r_2} IBrdr\mathbf{a}_z = IB\frac{r_2^2 - r_1^2}{2}\mathbf{a}_z$$

Force and Torque Calculation from Energy and Coenergy

A Singly Excited Linear Actuator

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant *t*, we record that the terminal voltage applied to the excitation winding is *v*, the excitation winding current *i*, the position of the movable plunger *x*, and the force acting on the plunger **F** with the reference

A singly excited linear actuator

direction chosen in the positive direction of the *x* axis, as shown in the diagram. After a time interval *dt*, we notice that the plunger has moved for a distance *dx* under the action of the force **F**. The mechanical done by the force acting on the plunger during this time interval is thus

$$dW_m = Fdx$$

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dW_e = dW_f + dW_m = vidt - Ri^2 dt$$

Because

$$e = \frac{d\lambda}{dt} = v - Ri$$

we can write

$$dW_f = dW_e - dW_m = eidt - Fdx$$
$$= id\lambda - Fdx$$

From the above equation, we know that the energy stored in the magnetic field is a function of the flux linkage of the excitation winding and the position of the plunger. Mathematically, we can also write
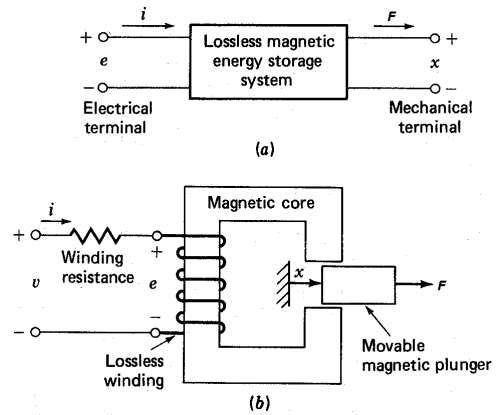
$$dW_f(\lambda, x) = \frac{\partial W_f(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_f(\lambda, x)}{\partial x} dx$$

Therefore, by comparing the above two equations, we conclude

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad \text{and} \quad F = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

From the knowledge of electromagnetics, the energy stored in a magnetic field can be expressed as

$$W_f(\lambda, x) = \int_0^\lambda i(\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda,x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

$$F = -\frac{\partial W_f(\lambda,x)}{\partial x} = \frac{1}{2}\left[\frac{\lambda}{L(x)}\right]^2 \frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or λ-i curve. Mathematically, if we define the area underneath the magnetization curve as the **coenergy** (which does not exist physically), i.e.

$$W_f{}'(i,x) = i\lambda - W_f(\lambda,x)$$

we can obtain

$$dW_f{}'(i,x) = \lambda\,di + i\,d\lambda - dW_f(\lambda,x)$$
$$= \lambda\,di + F\,dx$$
$$= \frac{\partial W_f{}'(i,x)}{\partial i}di + \frac{\partial W_f{}'(i,x)}{\partial x}dx$$

Energy and coenergy

Therefore,

$$\lambda = \frac{\partial W_f{}'(i,x)}{\partial i} \qquad \text{and} \qquad F = \frac{\partial W_f{}'(i,x)}{\partial x}$$
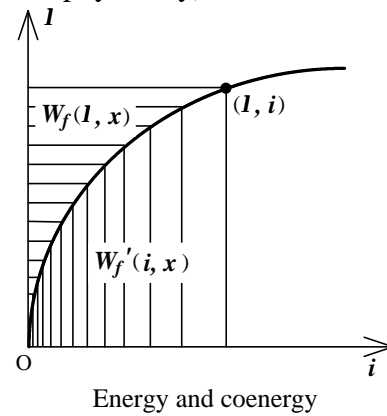
From the above diagram, the coenergy or the area underneath the magnetization curve can be calculated by

$$W_f{}'(i,x) = \int_0^i \lambda(i,x)\,di$$

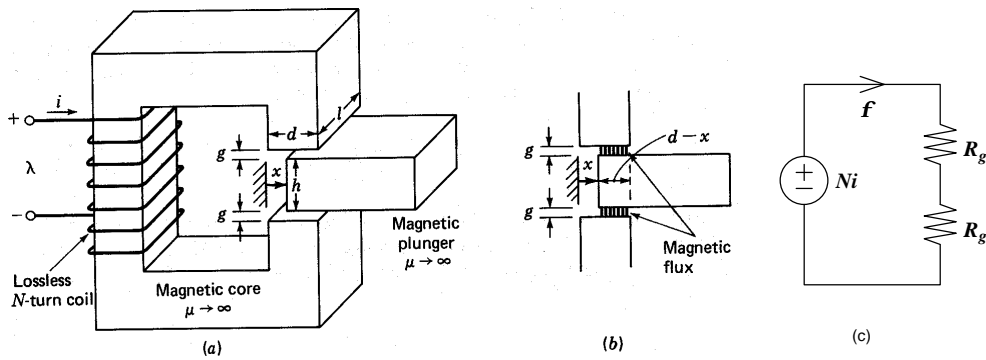For a magnetically linear system, the above expression becomes

$$W_f{}'(i,x) = \frac{1}{2}i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f{}'(i,x)}{\partial x} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

Example:

Calculate the force acting on the plunger of a linear actuator discussed in this section.

A singly excited linear actuator

Solution:

Assume the permeability of the magnetic core of the actuator is infinite, and hence the system can be treated as magnetically linear. From the equivalent magnetic circuit of the actuator shown in figure (c) above, one can readily find the self inductance of the excitation winding as

$$L(x) = \frac{N^2}{2R_g} = \frac{m_o N^2 l(d-x)}{2g}$$

Therefore, the force acting on the plunger is

$$F = \frac{1}{2}i^2 \frac{dL(x)}{dx} = -\frac{m_o l}{4g}(Ni)^2$$

The minus sign of the force indicates that the direction of the force is to reduce the displacement so as to reduce the reluctance of the air gaps. Since this force is caused by the variation of magnetic reluctance of the magnetic circuit, it is known as the **reluctance force**.

Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram on the right hand side. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the coenergy against the angular displacement, as summarized in the following table.
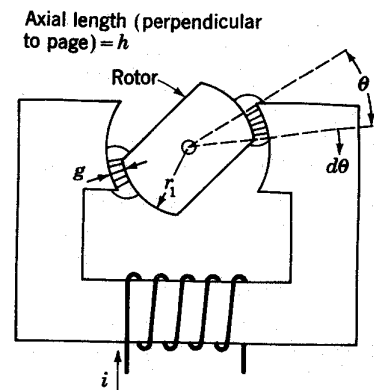
A singly excited rotating actuator

Table: Torque in a singly excited rotating actuator

Energy	**Coenergy**

In general,

$$dW_f = id\boldsymbol{l} - Td\boldsymbol{q} \qquad\qquad dW_f' = \boldsymbol{l}\,di + Td\boldsymbol{q}$$

$$W_f(\boldsymbol{l},\boldsymbol{q}) = \int_0^{\boldsymbol{l}} i(\boldsymbol{l},\boldsymbol{q})d\boldsymbol{l} \qquad\qquad W_f'(i,\boldsymbol{q}) = \int_0^{i} \boldsymbol{l}(i,\boldsymbol{q})di$$

$$i = \frac{\partial W_f(\boldsymbol{l},\boldsymbol{q})}{\partial \boldsymbol{l}} \qquad\qquad \boldsymbol{l} = \frac{\partial W_f'(i,\boldsymbol{q})}{\partial i}$$

$$T = -\frac{\partial W_f(\boldsymbol{l},\boldsymbol{q})}{\partial \boldsymbol{q}} \qquad\qquad T = \frac{\partial W_f'(i,\boldsymbol{q})}{\partial \boldsymbol{q}}$$

If the permeability is a constant,

$$W_f(\boldsymbol{l},\boldsymbol{q}) = \frac{1}{2}\frac{\boldsymbol{l}^2}{L(\boldsymbol{q})} \qquad\qquad W_f'(i,\boldsymbol{q}) = \frac{1}{2}i^2 L(\boldsymbol{q})$$

$$T = \frac{1}{2}\left[\frac{\boldsymbol{l}}{L(\boldsymbol{q})}\right]^2 \frac{dL(\boldsymbol{q})}{d\boldsymbol{q}} = \frac{1}{2}i^2\frac{dL(\boldsymbol{q})}{d\boldsymbol{q}} \qquad\qquad T = \frac{1}{2}i^2\frac{dL(\boldsymbol{q})}{d\boldsymbol{q}}$$

Doubly Excited Rotating Actuator

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and coenergy functions can be derived as following:
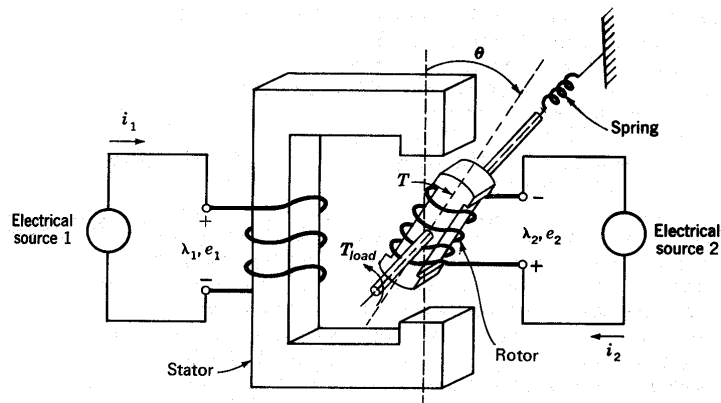
$$dW_f = dW_e - dW_m$$

where

$$dW_e = e_1 i_1 dt + e_2 i_2 dt$$

$$e_1 = \frac{d\boldsymbol{l}_1}{dt}, \qquad e_2 = \frac{d\boldsymbol{l}_2}{dt}$$

and

$$dW_m = Td\boldsymbol{q}$$

Hence,

A doubly excited actuator

$$dW_f(\lambda_1,\lambda_2,q) = i_1 d\lambda_1 + i_2 d\lambda_2 - T dq$$

$$= \frac{\partial W_f(\lambda_1,\lambda_2,q)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1,\lambda_2,q)}{\partial \lambda_2} d\lambda_2$$

$$+ \frac{\partial W_f(\lambda_1,\lambda_2,q)}{\partial q} dq$$

and

$$dW_f'(i_1,i_2,q) = d\left[i_1\lambda_1 + i_2\lambda_2 - W_f(\lambda_1,\lambda_2,q)\right]$$

$$= \lambda_1 di_1 + \lambda_2 di_2 + T dq$$

$$= \frac{\partial W_f'(i_1,i_2,q)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1,i_2,q)}{\partial i_2} di_2$$

$$+ \frac{\partial W_f'(i_1,i_2,q)}{\partial q} dq$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = -\frac{\partial W_f(\lambda_1,\lambda_2,q)}{\partial q}$$

or

$$T = \frac{\partial W_f'(i_1,i_2,q)}{\partial q}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix} = \begin{bmatrix}L_{11} & L_{12}\\L_{21} & L_{22}\end{bmatrix}\begin{bmatrix}i_1\\i_2\end{bmatrix}$$

or

$$\begin{bmatrix}i_1\\i_2\end{bmatrix} = \begin{bmatrix}\Gamma_{11} & \Gamma_{12}\\\Gamma_{21} & \Gamma_{22}\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix}$$

where $L_{12}=L_{21}$, $G_{11}=L_{22}/D$, $G_{12}=G_{21}=-L_{12}/D$, $G_{22}=L_{11}/D$, and $D=L_{11}L_{22}-L_{12}^2$. The magnetic energy and coenergy can then be expressed as

$$W_f(\lambda_1,\lambda_2,q) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1,i_2,q) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1 i_2$$

respectively, and it can be shown that they are equal.

Therefore, the torque acting on the rotor can be calculated as

$$T = -\frac{\partial W_f\left(\lambda_1, \lambda_2, q\right)}{\partial q} = \frac{\partial W_f{}'\left(i_1, i_2, q\right)}{\partial q}$$

$$= \frac{1}{2}i_1^2\frac{dL_{11}(q)}{\partial q} + \frac{1}{2}i_2^2\frac{dL_{22}(q)}{\partial q} + i_1 i_2\frac{dL_{12}(q)}{\partial q}$$

Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that $dL_{11}/dq \neq 0$. Similarly, the second term on the right hand side of the above torque express is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

Model of Electromechanical Systems

To illustrate the general principle for modeling of an electromechanical system, we still use the doubly excited rotating actuator discussed above as an example. For convenience, we plot it here again. As discussed in the introduction, the mathematical model of an electromechanical system consists of circuit equations for the electrical subsystem and force or torque balance equations for the mechanical subsystem, whereas the interactions between the two subsystems via the magnetic field can be expressed in terms of the *emf*'s and the electromagnetic force or torque. Thus, for the doubly excited rotating actuator, we can write

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + \frac{d\left(\lambda_{11} + \lambda_{12}\right)}{dt}$$

$$= R_1 i_1 + L_{11}\frac{di_1}{dt} + i_1\frac{dL_{11}(q)}{dq}\frac{dq}{dt} + L_{12}\frac{di_2}{dt} + i_2\frac{dL_{12}(q)}{dq}\frac{dq}{dt}$$

$$= \left[R_1 + w_r\frac{dL_{11}(q)}{dq}\right]i_1 + w_r\frac{dL_{12}(q)}{dq}i_2 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

A doubly excited actuator
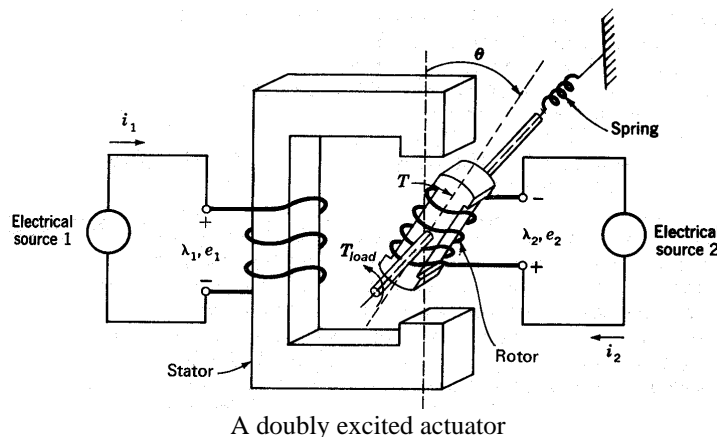
$$v_2 = R_2 i_2 + \frac{d\boldsymbol{l}_2}{dt} = R_2 i_2 + \frac{d\left(\boldsymbol{l}_{21} + \boldsymbol{l}_{22}\right)}{dt}$$

$$= R_2 i_2 + L_{12}\frac{di_1}{dt} + i_1 \frac{dL_{12}(\boldsymbol{q})}{d\boldsymbol{q}}\frac{d\boldsymbol{q}}{dt} + L_{22}\frac{di_2}{dt} + i_2 \frac{dL_{22}(\boldsymbol{q})}{d\boldsymbol{q}}\frac{d\boldsymbol{q}}{dt}$$

$$= \boldsymbol{w}_r \frac{dL_{12}(\boldsymbol{q})}{d\boldsymbol{q}} i_1 + \left[R_2 + \boldsymbol{w}_r \frac{dL_{22}(\boldsymbol{q})}{d\boldsymbol{q}}\right] i_2 + L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

and

$$T - T_{load} = J\frac{d\boldsymbol{w}_r}{dt}$$

where

$$\boldsymbol{w}_r = \frac{d\boldsymbol{q}}{dt}$$

is the angular speed of the rotor, T_{load} the load torque, and *J* the inertia of the rotor and the mechanical load which is coupled to the rotor shaft.

The above equations are nonlinear differential equations which can only be solved numerically. In the format of state equations, the above equations can be rewritten as

$$\frac{di_1}{dt} = -\frac{1}{L_{11}}\left[R_1 + \frac{dL_{11}(\boldsymbol{q})}{d\boldsymbol{q}}\boldsymbol{w}_r\right]i_1 - \frac{1}{L_{11}}\frac{dL_{12}(\boldsymbol{q})}{d\boldsymbol{q}}\boldsymbol{w}_r i_2 - \frac{L_{12}}{L_{11}}\frac{di_2}{dt} + \frac{1}{L_{11}}v_1$$

$$\frac{di_2}{dt} = -\frac{1}{L_{22}}\frac{dL_{12}(\boldsymbol{q})}{d\boldsymbol{q}}\boldsymbol{w}_r i_1 - \frac{1}{L_{22}}\left[R_2 + \frac{dL_{22}(\boldsymbol{q})}{d\boldsymbol{q}}\boldsymbol{w}_r\right]i_2 - \frac{L_{12}}{L_{22}}\frac{di_1}{dt} + \frac{1}{L_{22}}v_2$$

$$\frac{d\boldsymbol{w}_r}{dt} = \frac{1}{J}T - \frac{1}{J}T_{load}$$

and $\quad \dfrac{d\boldsymbol{q}}{dt} = \boldsymbol{w}_r$

Together with the specified initial conditions (the state of the system at time zero in terms of the state variables):

$$i_1\big|_{t=0} = i_{10}, \quad i_2\big|_{t=0} = i_{20}, \quad \boldsymbol{w}_r\big|_{t=0} = \boldsymbol{w}_{r0}, \quad \text{and} \quad \boldsymbol{q}\big|_{t=0} = \boldsymbol{q}_0,$$

the above state equations can be used to simulate the dynamic performance of the doubly excited rotating actuator.

Following the same rule, we can derive the state equation model of any electro-mechanical systems.

Exercises

1. An electromagnet in the form of a U shape has an air gap, between each pole and an armature, of 0.05 cm. The cross sectional area of the magnetic core is 5 cm^2 and it is uniformly wound with 100 turns. Neglecting leakage and fringing flux, calculate the current necessary to give a force of 147.2 N on the armature. Assume 15% of the total mmf is expended on the iron part of the magnetic circuit.

    Answer: 5.69 A

2. For a singly excited elementary two pole reluctance motor as shown in Fig.Q2, sketch the variation of the winding inductance with respect to the rotor position *q* and show that the magnitude of the average torque is

    $$T_{av} = 0.125 I_p^2 \left(L_d - L_q \right) \sin 2q_o$$

    where I_p is the peak value of the current, q_o the angle between the rotor and the stator at time zero and L_d and L_q are the inductances in the *d* and *q* axes respectively. Assume the excitation is sinusoidal and the instantaneous rotor position is *q=wt-q$_o$*.

3. For a singly excited elementary two pole reluctance motor under constant current conditions, calculate the maximum torque developed if the rotor radius is equal to 4 cm, the length of the air gap between a pole and the rotor equal to 0.25 cm, the axial length of the rotor equal to 3 cm. The pole excitation is provided by a coil of 1000 turns carrying 5 A.

    Answer: 3.77 Nm

4. Fig.Q4 shows a solenoid where the core section is square. Calculate the magnitude of the force on the plunger, given that *I*=10 A, *N*=500 turns, *g*=5 mm, *a*=20 mm, and *b*=2 mm.

    Answer: 256.4 N
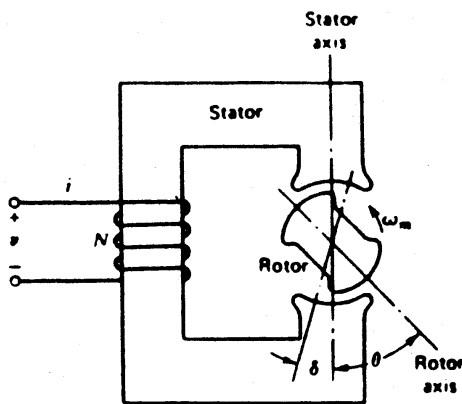
Fig.Q2
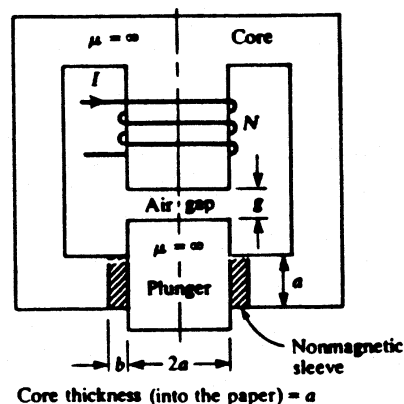
Fig.Q4

5. Calculate the self inductance of each coil in Fig.Q5 and the mutual inductance *M* between the coils. Is *M* positive or negative? Make appropriate assumptions.

6. Calculate the self inductance of the coil in Fig.Q6, making appropriate assumptions. Under what conditions could the iron reluctance be neglected?

7. In Fig.Q7, derive an expression for the flux f_{21} linking coil 2 due to a current in coil 1, making appropriate assumptions. Calculate the mutual inductance between the coils by two methods, and calculate the self inductance of each coil. Sketch f_{21} vs x.

8. Repeat question 7 using Fig.Q8, and sketch f_{21} vs q_1.

9. Repeat question 8 using Fig.Q9. Calculate also the mutual inductance between coils 1 and 3.

10. In Fig.Q10, the flux density is in the Z direction and is given by

$$B = B_m \sin\frac{px}{t}$$

Calculate the flux linking the coil.

11. In Fig.Q11, the poles are shaped to give a sinusoidal flux density $B=B_m\cos q$ at the rotor surface. With appropriate assumptions, calculate the flux linking coil 2 when the current in coil 1 is I_1. Calculate the mutual inductance.

Permeability: μ   Depth: *b*
Mean length: *l*

Fig.Q5

Permeability: μ   Depth: *b*
Mean length: *l*

Fig.Q6

Rectangular coil
size: *t´l*
t > *a* and *l* > *b*

Permeability: μ   Depth: *b*
Mean length: *l*

Fig.Q7

Permeability: μ→∞   Depth: *b*
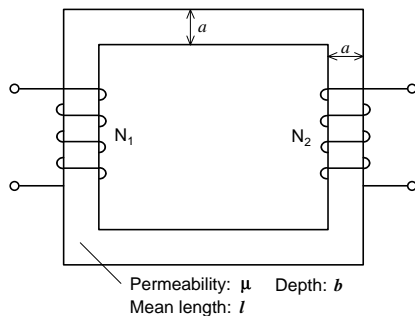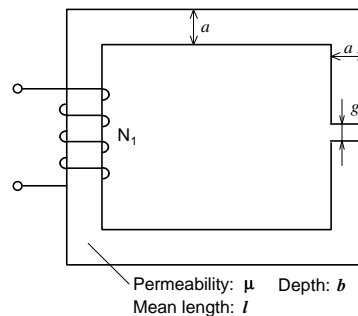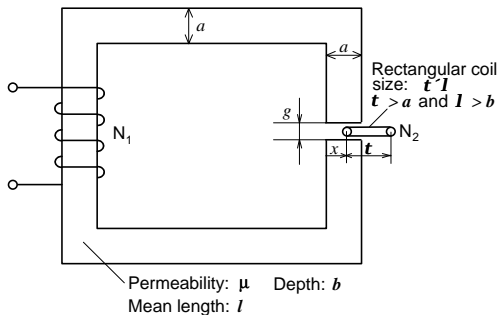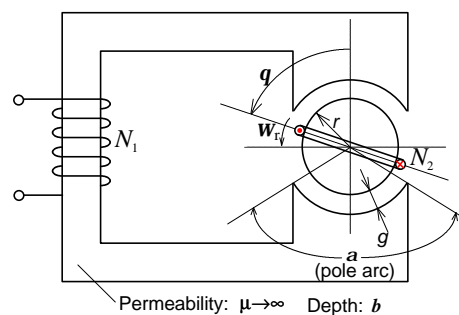
Fig.Q8
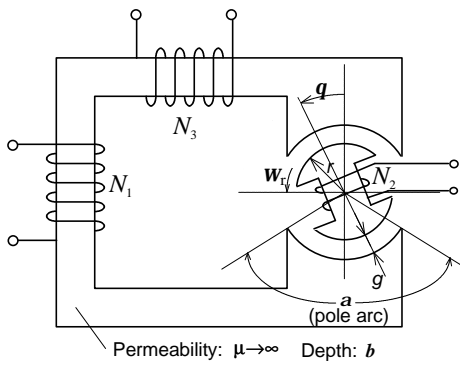
Fig.Q9
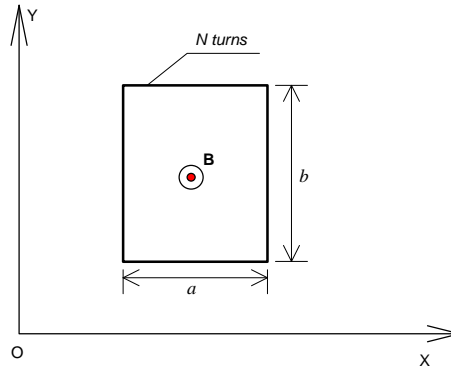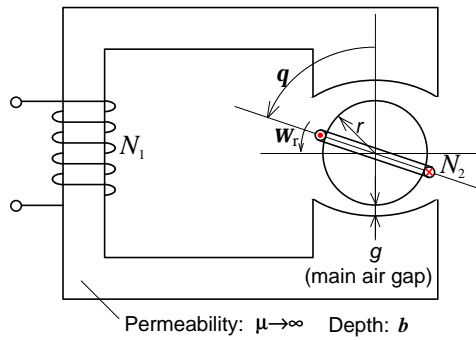
Fig.Q10

Fig.Q11

12. (a) Calculate the emf in the rectangular coil shown in Fig.Q12, if it moves in the X direction with
    velocity v and the flux density $B=B_o\sin bx$.
    (b) What value (or values) of the length a gives the maximum emf?
    (c) Calculate the emf if $B=B_o\sin(wt-bx)$ and the coil moves in the X direction with velocity v.
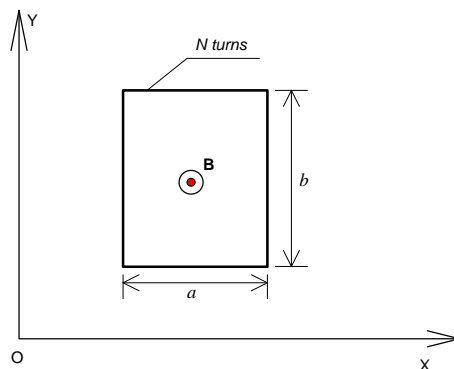   (d) What is the emf in (c) when $v=w/b$?

Fig.Q12

13. The diagram in Fig.Q13 show a cross section in the direction of motion of a linear machine (or a developed rotary machine) with (a) a smooth rotor and (b) a slotted rotor. The flux density shown is the average flux density in the Y direction near the

rotor surface. Derive expressions for the emf induced in the single coils shown when the rotor moves with velocity *v* in the X direction, and plot the emf against time.
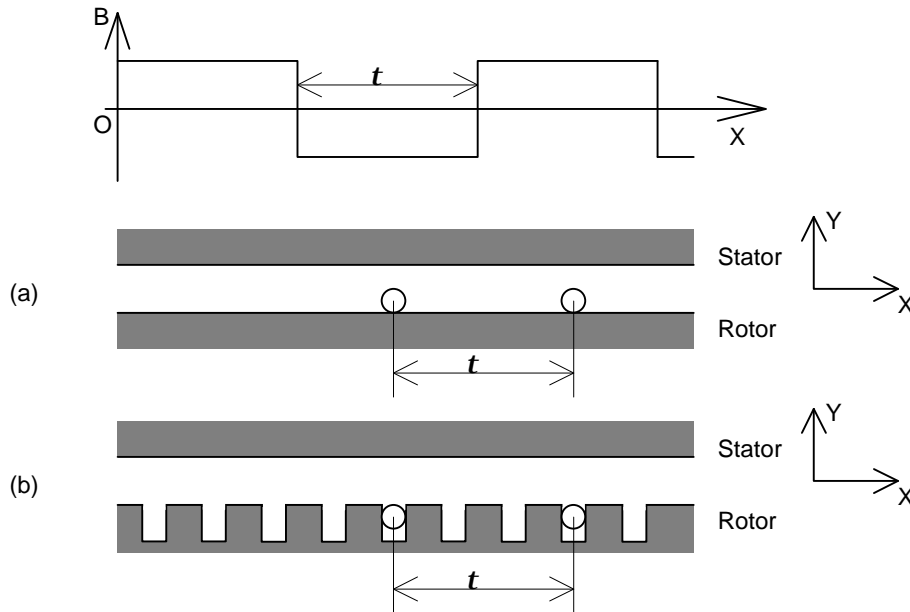
Fig.Q13

14. Derive an expression for the current induced in the ring shown in Fig.Q14, neglecting the flux produced by the current itself.

15. Derive an expression for the emf generated in the "Faraday disc" homopolar machine shown in Fig.Q15. Assume the flux density B is uniform and perpendicular to the disc between the brushes.
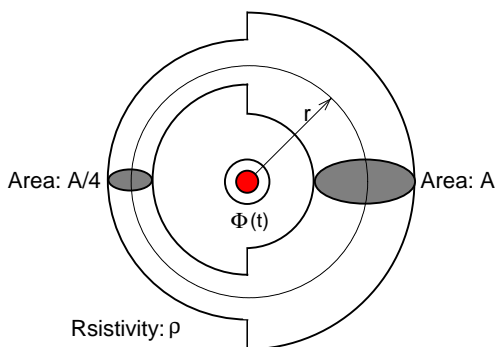

Fig.Q14
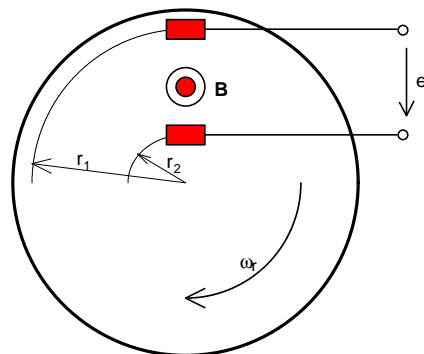
Fig.Q15

16. Derive an expression for the emf in coil in Fig.Q16 and sketch the emf vs time
    (a) when $i_2=0$ and $i_1=I_1$ a constant,
    (b) when $i_1=0$ and $i_2=I_2$ a constant,
    (c) when $i_2=0$ and $i_1=I_m\sin wt$,
    (d) when $i_1=I_1$ a constant and $i_2=I_m\sin wt$.
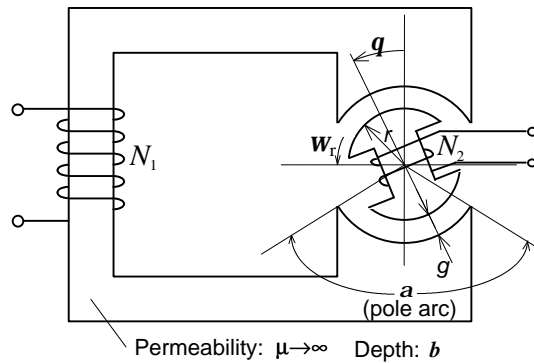   In parts (c) and (d), what happens if the rotor speed $w_r=w$? Explain.

Fig.Q16

17. Derive an expression for the emf in coil 2 in Fig.Q17 and sketch the emf vs. time, when
    (a) $i_1 = I_1$ a constant,
    (b) $i_1 = I_m \sin wt$.
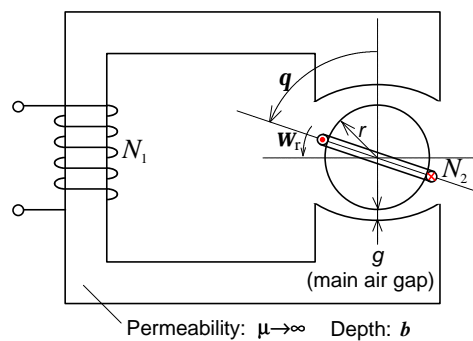   If the rotor coil is sunk into small slots, is the emf still the same? Explain.

18. In Fig.Q18 the poles are shaped to give a sinusoidal flux density $B = B_m \cos q$ at the rotor surface. Derive an expression for the emf in coil 2 and sketch the emf vs. time, when
    (a) $i_1 = I_1$ a constant,
    (b) $i_1 = I_m \sin wt$.
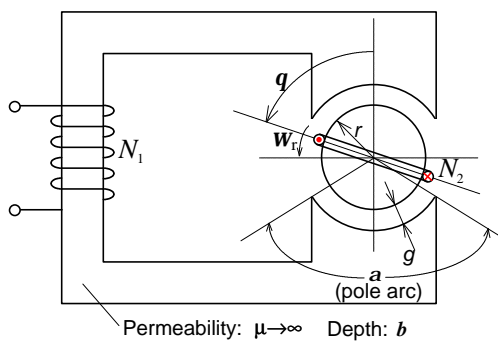
Fig.Q17   Fig.Q18

19. Find the torque on the rotating iron part in Fig.Q19 when the exciting winding has *NI* A-turns, the length of both air gaps is *g* and the reluctance of the rest of the magnetic circuit is negligible. Neglect fringing effects. The length of the machine is *b* and the rotor diameter *d*.

Answer: $T = \dfrac{1}{2} m_o \dfrac{bd}{4g} (NI)^2$

20. Fig.Q20 shows a straight conductor moving between two iron poles of length *b*. What is the force in the position shown if the only flux is that produced by the current I_c in the conductor? Assume the ratio *g/a* is small, and the permeability of the iron is infinite.

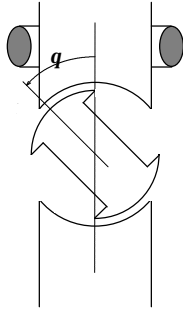*Answer:* $F_x = \dfrac{1}{2} m_o \dfrac{b(a-2x)}{ga} I_c^2$

Fig.Q19
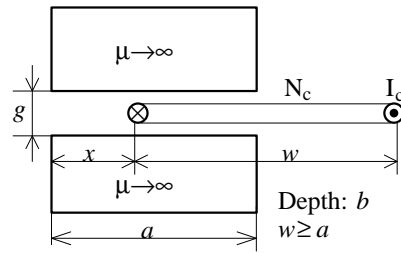

Fig.Q20

21. If the poles of Fig.Q20 are closed as shown in Fig.Q21, calculate the force on the conductor.

Answer: $F_x = \dfrac{1}{2} \boldsymbol{m}_o \dfrac{b}{g} I_c^2$

22. If a coil of *N* turns carrying a current *I* is wound on the yoke of Fig.Q21, as shown in Fig.Q22, calculate the force on the moving conductor. Is the force given by the product of the current in the conductor and flux density produced by the coil?

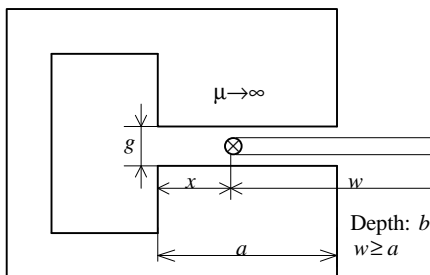*Answer:* $F_x = \boldsymbol{m}_o \dfrac{b}{g}\left(\dfrac{1}{2} I_c + NI\right) I_c$, *No*
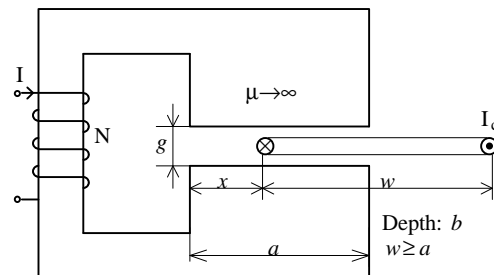

Fig.Q21

Fig.Q22

23. In the relay (or loudspeaker) shown in Fig.Q23 the coil is circular and moves in a uniform annular gap of small length *g*. What is the force on the coil if the total current in coil cross section is I_c,
    (a) when the exciting winding carries no current, or
    (b) when the exciting winding ampere-turns are *NI*?
    Note that, in the latter case, the force is not given by the current in the coil times the flux density due to the exciting winding.

Answer: $F = \dfrac{1}{2} \boldsymbol{m}_o I_c^2 \dfrac{pd}{g}$, $\quad F = \boldsymbol{m}_o \dfrac{pd}{g}\left(\dfrac{1}{2} I_c + NI\right) I_c$

24. A machine of axial length *b* with a single rotor coil is shown in Fig.Q24. Calculate the torque when the rotor coil (of N_c turns carrying a current I_c) is placed
    (a) on the rotor surface and

(b) in small rotor slots.

Is the torque given by the product of the current in the rotor coil and the flux density produced by the stator coil?

Answer: $T = \dfrac{1}{2} \boldsymbol{m}_o \dfrac{bd}{g} N I N_c I_c$, *Yes*
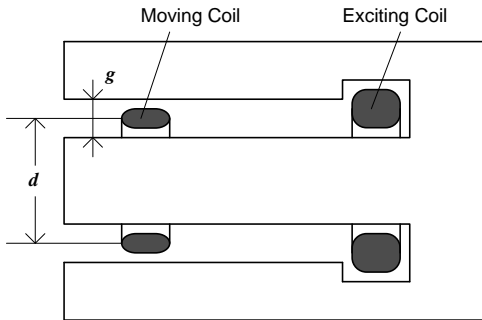

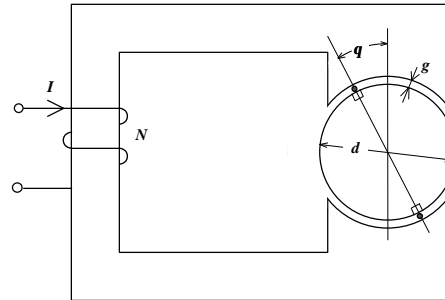Fig.Q23   Fig.Q24

25. The cylindrical iron core C in Fig.Q25 is constrained so as to move axially in an iron tube. A coil of *NI* A-turns is wound in a single peripheral slot as shown in the diagram. What is the axial force on the core and how does it vary with the position of the core (making any necessary assumptions)?

Answer: $F_x = \dfrac{1}{2} \boldsymbol{m}_o \dfrac{pd}{g} (NI)^2 \dfrac{a-2x}{a}$

26. Fig.Q26 shows in cross section the construction of a "motor" that gives a limited range of movement reversible according to the direction of current in a coil 1. The coil 2 is an exciting or polarizing winding, supplied from a constant source. Find the torque in the position shown, for the same rotor dimensions and assumptions as in problem 1, when the one coil has N_1I_1 A-turns and the other N_2I_2.

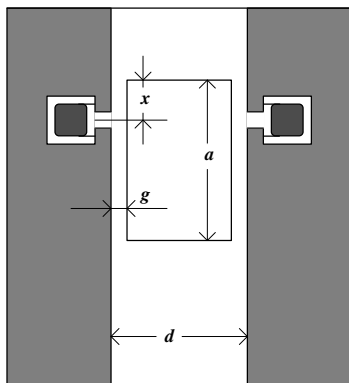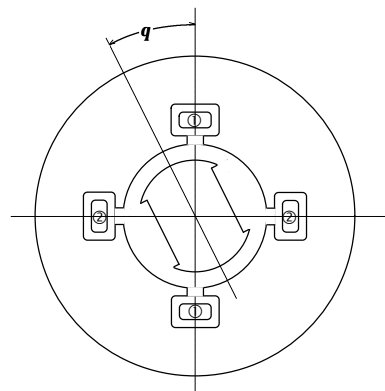*Answer:* $T = \dfrac{1}{2} \boldsymbol{m}_o \dfrac{bd}{g} N_1 I_1 N_2 I_2$

Fig.Q25   Fig.Q26